

Skill 1 : Sketch the angle for each of the following angle in separate cartesian planes

a) 520°	b) 1050°	d) $\frac{7}{2}\pi$ rad	e) $\frac{10}{3}\pi$ rad
f) -135°	g) -430°	h) $-\frac{\pi}{4}$ rad	i) $-\frac{7}{2}\pi$

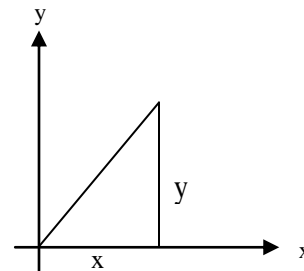
Skill 2 : Understand and use the six trigonometric functions of any angle

$$r = \sqrt{x^2 + y^2} .$$

$$\sin \theta = \frac{y}{r} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{x}{r}, \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{y}{x} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$



Skill 3 : Find values of the six trigonometric functions of any angle

<p>1. Given $\sin 45^\circ = 0.707$ and $\cos 45^\circ = 0.707$ Find the value of</p> <p>$\tan 45^\circ =$</p> <p>$\cot 45^\circ =$</p> <p>$\sec 45^\circ =$</p> <p>$\operatorname{cosec} 45^\circ =$</p>	<p>2. Given $\sin \frac{2}{3}\pi = 0.866$ and $\cos \frac{2}{3}\pi = -0.5$. Find the value of</p> <p>$\tan \frac{2}{3}\pi =$</p> <p>$\cot \frac{2}{3}\pi =$</p> <p>$\sec \frac{2}{3}\pi =$</p> <p>$\operatorname{cosec} \frac{2}{3}\pi =$</p>
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3. Given $\sin 15^\circ = 0.259$ and $\cos 15^\circ = 0.966$,
Find the value of

$$\tan 15^\circ =$$

$$\cot 15^\circ =$$

$$\sec 15^\circ =$$

$$\operatorname{cosec} 15^\circ =$$

4. Given $\sin \frac{4}{3}\pi = -0.866$ and $\cos \frac{4}{3}\pi = -0.5$
Find the value of

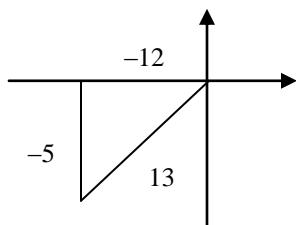
$$\tan \frac{4}{3}\pi =$$

$$\sec \frac{4}{3}\pi =$$

$$\cot \frac{4}{3}\pi =$$

$$\operatorname{cosec} \frac{4}{3}\pi =$$

5: Given that $\sin \theta = \frac{5}{13}$, $0^\circ < \theta < 270^\circ$ Find the value of each the following trigonometric function without using a calculator



From diagram:

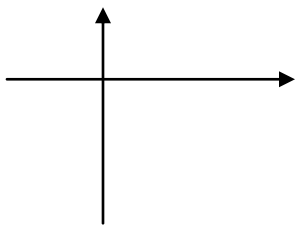
$$\sin \theta = -\frac{5}{13} \quad \cos \theta = \quad \tan \theta =$$

$$\text{a) } \cot \theta =$$

$$\text{b) } \operatorname{cosec} \theta =$$

$$\text{c) } \sec \theta =$$

6. Given that $\cos \theta = \frac{12}{13}$, $180^\circ < \theta < 360^\circ$. Find the value of each the following trigonometric function without using a calculator



From diagram:

$$\cos \theta = \frac{12}{13} \quad \sin \theta = \quad \tan \theta =$$

$$\text{a) } \cot \theta =$$

$$\text{b) } \operatorname{cosec} \theta =$$

$$\text{c) } \sec \theta =$$

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Exercise 5.**

Skill 4 : Solving trigonometric equations

FOLLOW THE STEP BELOW;

S1 : From the sign of the trigonometric ratio, determine the quadrants in which angle lie

S2 : Without considering the sign of the trigonometric ratio, find the reference angle using calculator

S3 : Sketch the reference angle in the quadrants

S4 : Determine all the possible solutions in the given range of the angles



Skill 4.1 : Solve each of the following trigonometric equation for $0^\circ \leq \theta \leq 360^\circ$

1.	<p>a. $\cos \theta = -0.9063$</p> <p>S1: $\cos \theta$ has a negative value in the second and third quadrants.</p> <p>S2 : the reference angle $= 25^\circ$</p> <p>S4: $\theta = 180^\circ - 25^\circ, 180^\circ + 25^\circ$ $= 155^\circ, 205^\circ$</p>	<p>S3</p>
2.	<p>$\cos \theta = -0.6428$</p> <p>[130°, 230°]</p>	<p>3. $\tan \theta = 0.5$</p> <p>[26°34', 206° 34']</p>
4.	<p>$\tan 2\theta = 1.732$</p> <p>[30°, 120°, 210°, 300°]</p>	<p>5. $2\tan \theta = 3$</p> <p>[56° 19', 236° 19']</p>

6	$\cos 2\theta = \frac{1}{2}$	$7. \tan(\theta + 60^\circ) = -1$
[$30^\circ, 150^\circ, 210^\circ, 330^\circ$]		[$75^\circ, 240^\circ$]
8	$\tan\left(\frac{1}{2}\theta - 15^\circ\right) = 0.8687$	$9. 2 \tan 3\theta = -1$
[$111^\circ 58'$]		[$51^\circ 7', 111^\circ 7', 171^\circ 7', 231^\circ 9', 291^\circ 9', 513^\circ 9'$]

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Skill 4.2 : Solve each of the following trigonometric equation for $0^\circ \leq \theta \leq 360^\circ$

$1. \sin \theta = -\cos 48^\circ$	$2. 5 \cos \theta \sin \theta = \cos \theta$	$3. 2\sin \theta = \cos \theta$
[$222^\circ, 318^\circ$]	[$11^\circ 32', 168^\circ, 28^\circ, 90^\circ, 270^\circ$]	[$26^\circ 34', 206^\circ 34'$]

Skill 4.3 : Solve each of the following trigonometric equation for $0^\circ \leq \theta \leq 360^\circ$

<p>1. $(1 + \sin x)(\cos^2 x) = 0$</p> <p>[$270^\circ, 225^\circ \dots$]</p>	<p>2. $6 \sin x + \operatorname{cosec} x = 5$</p> <p>[$19^\circ 28', 160^\circ 32', 30^\circ, 150^\circ$]</p>	<p>3. $2 \tan x - 1 = \cot x$</p> <p>[$153^\circ 26', 333^\circ 26', 45^\circ$]</p>
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Skill 4.4 : Solve each of the following trigonometric equation for $0^\circ \leq \theta \leq 360^\circ$

<p>1. $2\cos^2 x + 5 \cos x - 3 = 0$</p> <p>[$60^\circ, 300^\circ$]</p>	<p>2. $3\sin x = \tan x$</p> <p>[$0^\circ, 70^\circ 40', 289^\circ 20'$]</p>	<p>3. $2 \tan^2 x + \tan x - 3 = 0$</p> <p>[$45^\circ, 12' 303$]</p>
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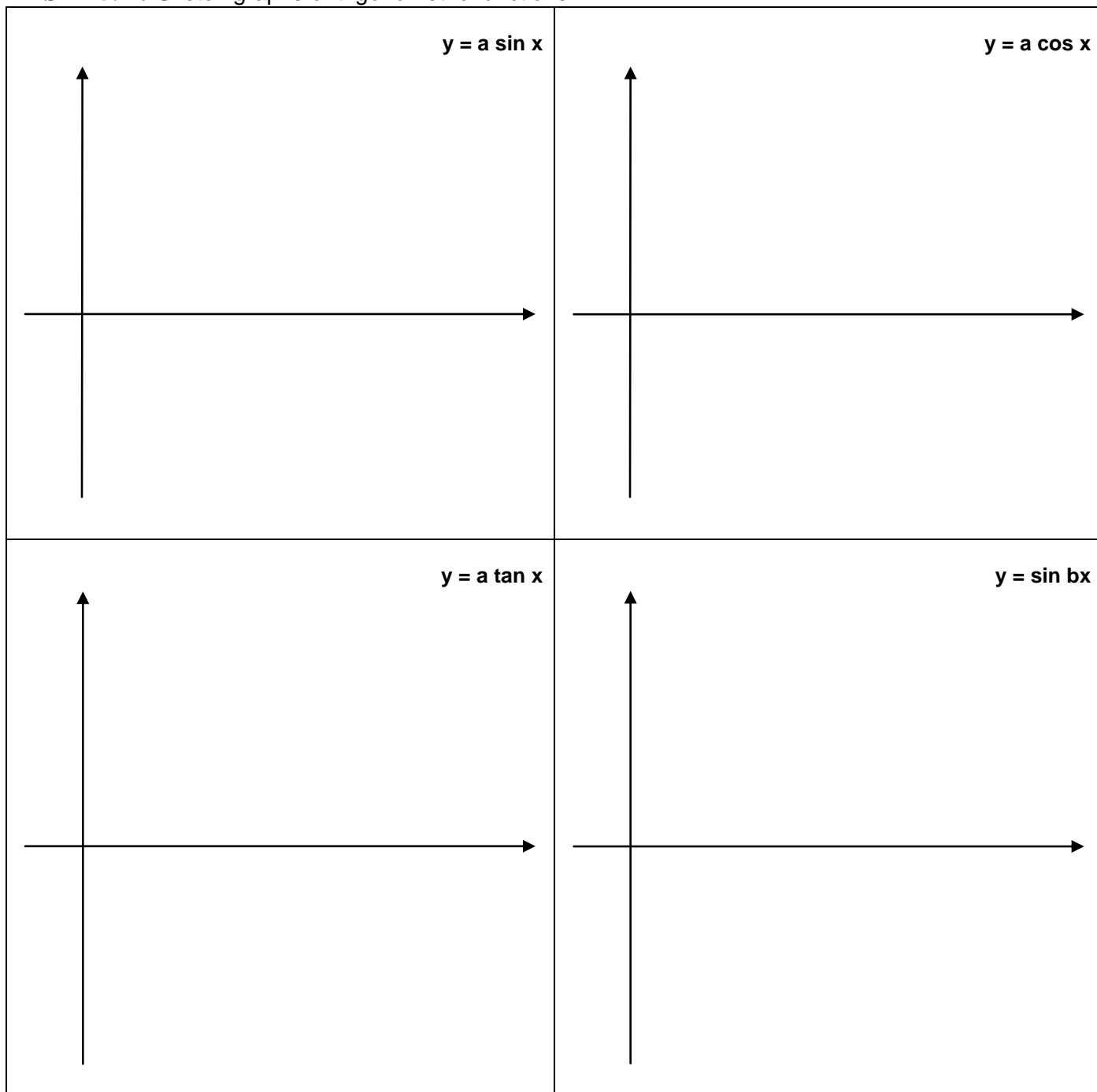
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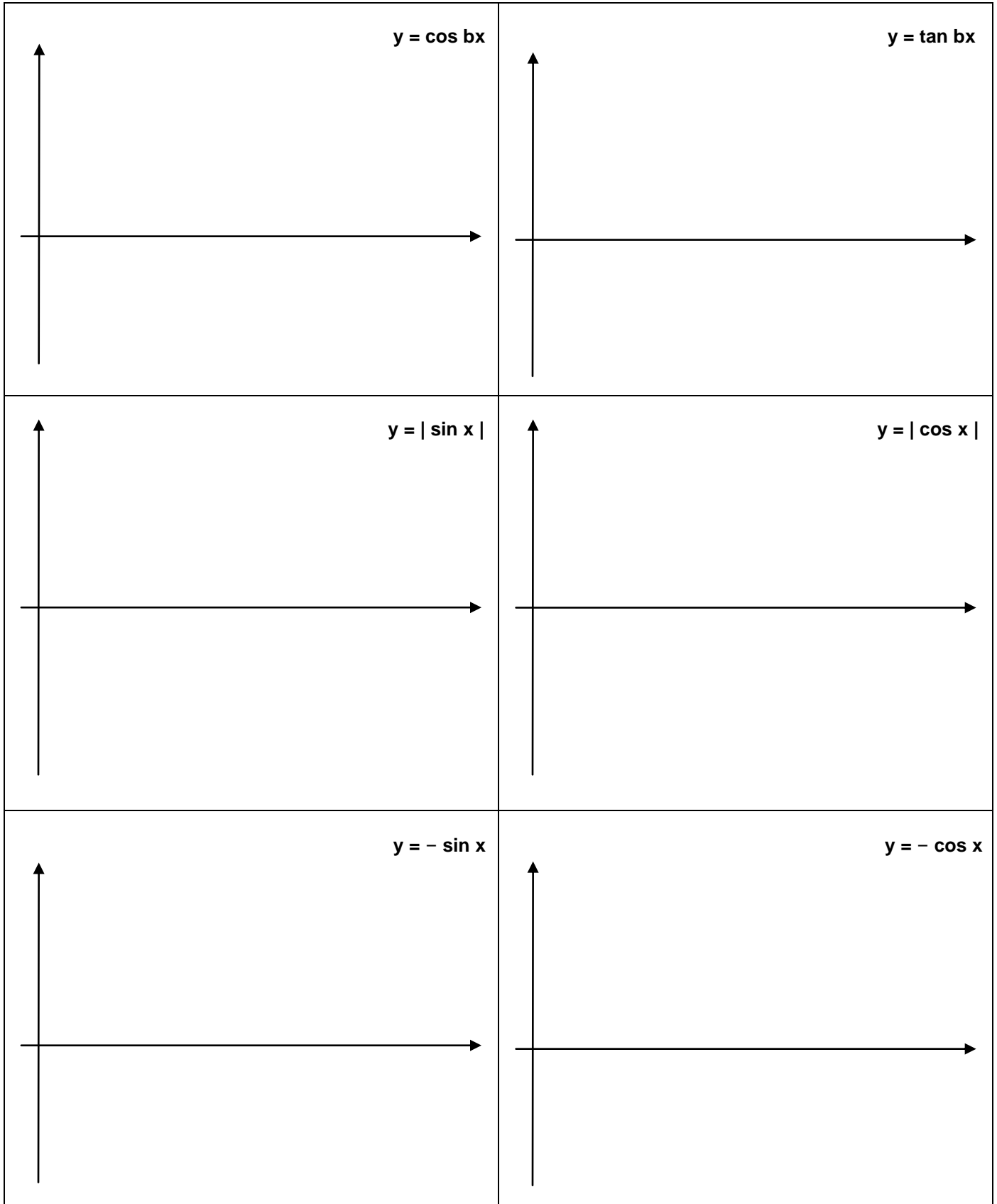
<p>1. Given $\sin x = \frac{p}{3}$ where x is a acute angle. Express $\cot x$ in terms of p</p> <p>[$\frac{\sqrt{9-p^2}}{p}$]</p>	<p>2. Solve the equation $4 \tan^2 x = 1$ for $90^\circ < x < 360^\circ$</p> <p>[$26^\circ 34', 153^\circ 26', 206^\circ 43', 333^\circ 26'$]</p>	<p>3. Solve the equation for $0^\circ < \theta < 360^\circ$</p> $6 \cos^2(\theta - \frac{\pi}{3}) - \cos(\theta - \frac{\pi}{3}) = 2$ <p>[$11.8^\circ, 108.2^\circ 180^\circ, 300^\circ$]</p>
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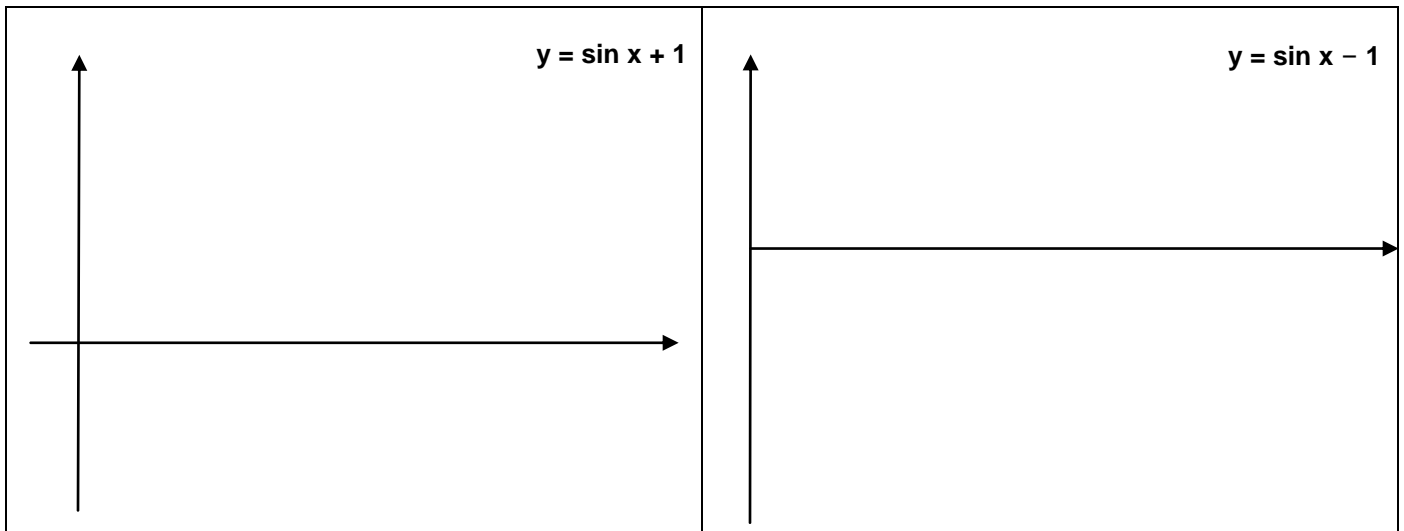
Skill 5 : Sketch graphs of trigonometric functions:

- $y = c + a \sin bx$,
 - $y = c + a \cos bx$,
 - $y = c + a \tan bx$,
- where a , b and c are constants and $b > 0$.

Skill 5.1 : Sketch graphs of trigonometric functions







Skill 5.2 : Determine the number of solutions to a trigonometric equation using sketched graphs.

<p>1. Sketch the graph of $y = 3 \sin 2x$ for $0^\circ \leq x \leq 360^\circ$. Determine the number of solution to the equation $3 \sin 2x + \frac{1}{2}x - 2 = 0$</p>	<p>2. Sketch the graph of $y = \tan x$ for $0 \leq x \leq 2\pi$. Determine the number of solution to the equation $\tan x = \frac{1}{3}x + 3 = 0$</p>
<p>3. Sketch the graph of $y = 3 \cos 2x$ for $0^\circ \leq x \leq 360^\circ$. Determine the number of solutions to the equation $3\pi \cos 2x - 2x = 0$</p>	<p>4. Sketch the graph of $y = 1 - 2\sin x$ for $0 \leq x \leq 2\pi$. Hence, draw a suitable straight line on the same axis to find the number of solutions to the equation $\pi - 2\pi \sin x = 3x$, for $0 \leq x \leq 2\pi$. State the number of solutions.</p>

Skill 6 : Basic identities.

<p>1. $\sin^2 x + \cos^2 x = 1$</p> <p>2. $\sec^2 x = 1 + \tan^2 x$</p> <p>3. $\operatorname{cosec}^2 x = 1 + \cot^2 x$</p> <p>4. $\tan x = \frac{\sin x}{\cos x}$</p> <p>5. $\sec x = \frac{1}{\cos x}$</p> <p>6. $\operatorname{cosec} x = \frac{1}{\sin x}$</p> <p>7. $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$</p>	<p>} Basic Identities.</p> <p>} Six Trigonometri Ratio</p>	<p>Guide to proving trigonometric identities</p> <p>1. Choose either right or left - hand side</p> <p>2. Use Basic Identities .or / and Six Trigonometri Ratio</p>
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Skill 6.1 : Prove each of the following trigonometric identities

a) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$	b) $\cot x \cos x = \operatorname{cosec} x - \sin x$	c) $\sin y + \cos^2 y \operatorname{cosec} y = \operatorname{cosec} y$
d) $\tan^2 x \sin^2 x = \tan^2 x \sin^2 x$	e) $\tan x + \cot x = \operatorname{cosec} x \sec x$	f) $\frac{1}{1 + \sin y} + \frac{1}{1 - \sin y} = 2\sec^2 y$

Skill 6.2 : Solve trigonometric equations using basic identities.

<p>a) $5 \sin^2 x - 2 = 2 \cos x$</p> <p>[53.13° 180°, 306.87°]</p>	<p>b) $4 \cos x - 3 \cot x = 0$</p> <p>[48.59°90°131.41°,270°]</p>	<p>c) $\tan^2 x + 8 = 7 \sec x$</p> <p>[48°11' , 60°,300° , 311°49']</p>
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SPM Questions

<p>a) Solve the equation $6 \cos x = 1 + 2 \sec x$ for $0 \leq x \leq 360^\circ$</p> <p>[48°11',120°,240°, 311° 49']</p>	<p>b) Solve the equation $2 \operatorname{cosec}^2 x = 7 + \frac{3}{\tan x}$ for $0 \leq x \leq 360^\circ$</p> <p>[21°48' , 135°,201°48' ,315°]</p>
<p>c) Solve the equation $4 \cos x = \sqrt{3} \operatorname{cosec} x$ for $0 \leq x \leq 360^\circ$</p> <p>[30°,60°,210°, 240°]</p>	<p>d) Solve the equation $5 \tan x - \cot y = \sec y$ for $0 \leq y \leq 360^\circ$</p> <p>[30°,150°,240°, 199° 28' , 340° 32']</p>

Skill 7 : Addition formulae and double-angle

Addition and double Angle Formulae

Addition Formulae	Double angle Formulae	Half-angle formulae.
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\sin 2A = 2 \sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A$ $= 2\cos^2 A - 1$ $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$	$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$ $\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$ $=$ $=$ $\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$

Skill 7.1 : Find the value of addition and double angle

<p>3. Given $\tan A = \frac{5}{12}$, $\cot B = \frac{4}{3}$ where A and B are acute angle. Find the value each of the following without using calculator</p> <p>a) $\sin(A - B)$ b) $\tan 2B$</p> <p>c) $\cos(A+B)$ d) $\tan \frac{A}{2}$</p> <p>Sketches</p>	<p>2. Given $\sin A = \frac{12}{13}$, $\cos B = -\frac{4}{5}$ where A and B are obtuse angle .Without using calculator find the value of</p> <p>a) $\cos(A - B)$ b) $\tan(A + B)$</p> <p>Sketches</p>
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Skill 7.2 : Prove trigonometric identities using addition formulae and/or double-angle formulae

<p>a) $\cot x - \operatorname{cosec} 2x = \cot 2x$</p>	<p>b) $\sin 2x = \frac{1 - \cos 2x}{\tan x}$</p>	<p>c) $2 \cot 2x + \tan x = \cot x$</p>
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Skill 7.3 : Solve trigonometric equation using addition formulae and/or double-angle formulae

<p>a) $3 \sin 2x = 2 \sin x$</p> <p>[$70^{\circ}32'$, 180°, $289^{\circ}28'$]</p>	<p>b) $3 \cos 2x - 7 \cos x = -5$</p> <p>[$48^{\circ}11'$, 60°, 300°, $311^{\circ}49'$]</p>	<p>c) $\tan 2x = 10 \tan x$</p> <p>[$41^{\circ}49'$, $138^{\circ}11'$, 180°, $221^{\circ}49'$, $318^{\circ}11'$]</p>
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SPM Questions

<p>1. Given $3 \tan 2x = 4$ for $90^\circ \leq x \leq 180^\circ$. Find the value of $\sin^2 x$</p> <p>[Ans 4/ 5]</p>	<p>2. Given $\sin(x - y) = \frac{1}{2}$ and $\cos x \sin y = \frac{3}{4}$ Find the value each of the following</p> <p>i) $\sin x \cos y$ ii) $\sin(x + y)$</p>
<p>3. Prove that $2 \cot 2x + \tan x = \cot x$</p>	<p>4. Given $\tan \theta = \frac{1}{3}$ without using calculator find the value of</p> <p>(i) $\tan 2\theta$ (ii) $\tan(135^\circ - \theta)$ [SPM 93]</p> <p style="text-align: right;">$\frac{3}{4}, -2$</p>
<p>5. Given θ is a acute angle and $\sin \theta = p$ express each of the following in term of p [SPM 94]</p> <p>(i) $\tan \theta$ (ii) $\cos(-\theta)$ (iii) $\cos 2\theta$</p> <p style="text-align: right;">$\frac{p}{\sqrt{1-p^2}}, \sqrt{1-p^2}, 1-2p^2$</p>	<p>6. Solve each of the following for $90^\circ \leq \beta \leq 270^\circ$ [SPM 95]</p> <p>i) $2 \tan^2 \beta = 1$ ii) $2 - 3 \sin \beta - \cos 2\beta = 0$</p> <p style="text-align: right;">(a) $144^\circ 44', 215^\circ 16'$ (b) $90^\circ, 150^\circ$</p>

<p>7. Given $\sin \theta = k$ where θ is a acute Angle, find</p> <p>i) $\sin 2\theta$ in term of k ii) the positive value of k if $\cos 2\theta = k$ [SPM 98]</p>	<p>8. Show $\frac{\sin 2\theta + \sin \theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$ [SPM 97]</p>
<p>9. Solve the equation $4\sin(x - \pi) \cos(x - \pi) = 1$ for $0 \leq x \leq 2\pi$ [SPM 97]</p> <p style="text-align: right;">$1\frac{1}{12}\pi, 1\frac{5}{12}\pi$</p>	<p>10. Given $\tan 2y = \frac{5}{12}$ where $90^\circ \leq y \leq 180^\circ$. Find the value of $\cos^2 y$ [SPM 97]</p> <p style="text-align: right;">$\frac{1}{26}$</p>
<p>11. Prove that $\tan^2 x - \cot^2 x = \sec^2 x - \operatorname{cosec}^2 x$ [SPM 98]</p>	<p>12. Draw the graph of $y = 2 \cos \frac{\pi}{2} x$, for $0 \leq x \leq 3$. Hence , draw a suitable straight line on the same axis to find the values of x satisfying the equation $\cos \frac{\pi}{2} x + \frac{1}{4} = 0$ for $0 \leq x \leq 3$. [SPM 98]</p> <p style="text-align: right;">1.15 and 2.85</p>