

## Skill 1 : Indefinite Integral

### Skill 1.1 : Integrals of $x^n$ :

1. $\int x^3 dx = \frac{x^{3+1}}{3+1} + c$ $= \frac{x^4}{4} + c$	2. $\int x^5 dx =$	3. $\int x^9 dx =$
4. $\int x^{-3} dx =$	5. $\int x^{-2} dx =$	6. $\int x dx =$

### Skill 1.2 : Integral of $ax^n$ :

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$

Note :  $\int m dx = mx + c$  ,  $c$  is a constant

1. $\int 10 dx = 10x + c$	2. $\int \frac{1}{2} dx =$	3. $\int -3 dx =$
4. $\int 10x^4 dx =$	5. $\int 4x^3 dx =$	6. $\int 6x^3 dx = 6 \cdot \frac{x^{3+1}}{3+1} + c$ $= 6 \cdot \frac{x^4}{4} + c$ $= \frac{3x^4}{2} + c$
7. $\int 8x dx = 8 \cdot \frac{x^{1+1}}{1+1} + c$ $= 8 \cdot \frac{x^2}{2} + c$ $= 4x^2 + c$	8. $\int 6x dx =$	9. $\int 3x dx =$
10. $\int 12x^3 dx =$	11. $\int 8x^2 dx =$	12. $\int 10x^5 dx =$

$13. \int \frac{2}{x^3} dx = \int 2x^{-3} dx$ $= 2 \cdot \frac{x^{-3+1}}{-3+1} + c$ $= 2 \cdot \frac{x^{-2}}{-2} + c$ $= -\frac{1}{x^2} + c$	$14. \int \frac{8}{x^5} dx = \int 8x^{-5} dx$ $=$	$15. \int \frac{12}{x^4} dx =$
$16. \int \frac{2}{5x^3} dx =$	$17. \int \frac{2}{3} x dx =$	$18. \int 0.9x^2 dx =$

### Skill 1.3 : Integral of Algebraic Terms

Note : Integrate term by term. Expand & simplify the given expression where necessary.

Example :  $\int (3x^2 - 4x + 5)dx = \frac{3x^3}{3} - \frac{4x^2}{2} + 5x + c = x^3 - 2x^2 + 5x + c$

$1. \int (6x - 4)dx$ $=$	$2. \int (12x^2 + 8x - 1)dx$ $=$	$3. \int (x^3 - 3x + 2)dx$ $=$
$4. \int x(3x - 2)dx$ $=$	$5. \int (2x - 1)(2x+1)dx$ $=$	$6. \int (x+2)(x-3)dx$ $=$

7. $\int (3x - 2)^2 dx$ =	8. $\int \frac{(2x - 1)(2x + 1)}{x^2} dx$ =	9. $\int \frac{6x^2 - 4}{x^2} dx$ =
10. $\int (3 - \frac{1}{x})(3 + \frac{1}{x}) dx$ =	11. $\int (-2x^{-2} + 3 - x + \frac{1}{x^2}) dx$ =	12. $\int x(3 - x)^2 dx$ =

**Skill 1.4 : Integral of  $\int (ax + b)^n dx, n \neq -1$**

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, n \neq -1$$

EXAMPLE	$\int (3x + 2)^4 dx = \frac{(3x + 2)^{4+1}}{3(4+1)} + c$ $= \frac{(3x + 2)^5}{15} + c$	EXAMPLE	$\int \frac{12}{(2x - 3)^4} dx = \int 12(2x - 3)^{-4} dx$ $= \frac{12(2x - 3)^{-3}}{-3 \cdot (2)} + c$ =
1.	$\int (2 + 4x)^5 dx =$	2.	$\int (x + 2)^{-4} dx =$
3.	$\int \frac{3}{(2x - 1)^2} dx =$	4.	$\int \frac{15}{(3x - 5)^6} dx =$

5.	$\int 6(2-x)^3 dx =$	6.	$\int 30(4+3x)^{-3} dx =$
7.	$\int \frac{2}{3}(1-2x)^3 dx =$	8.	$\int \frac{15}{2(x-3)^4} dx =$

**Skill 1.5 : Determine the equation of curve from gradient function**

1	<p>Given <math>\frac{dy}{dx} = 2x + 2</math> and <math>y = 6</math> when <math>x = -1</math>, find <math>y</math> in terms of <math>x</math>. SPM 2003, K2</p> $\frac{dy}{dx} = 2x + 2$ $y = \int (2x + 2) dx$ $= \frac{2x^2}{2} + 2x + c$ $y = x^2 + 2x + c$ <p><math>y = 6, x = 1,</math> <math>6 = 1^2 + 2(1) + c</math></p> $6 = 3 + c$ $c = 3$ <p>Hence <math>y = x^2 + 2x + 3</math></p>	2	<p>Given <math>\frac{dy}{dx} = 2x + 3</math> and <math>y = 4</math> when <math>x = 1</math>, find <math>y</math> in terms of <math>x</math>.</p> $y = x^2 + 3x$
3	<p>Given <math>\frac{dy}{dx} = 4x + 1</math> and <math>y = 4</math> when <math>x = -1</math>, find <math>y</math> in terms of <math>x</math>.</p>	4	<p>Given <math>\frac{dy}{dx} = 6x - 3</math> and <math>y = 3</math> when <math>x = 2</math>, find <math>y</math> in terms of <math>x</math>.</p>

			$y = 2x^2 + x + 3$		$y = 3x^2 - 3x - 3$
5	Given $\frac{dy}{dx} = 4 - 2x$ and $y = 5$ when $x = 1$ , find $y$ in terms of $x$ .	6	Given $\frac{dy}{dx} = 3x^2 - 2$ and $y = 4$ when $x = -1$ , find $y$ in terms of $x$ .		
			$y = 4x - x^2 + 2$		$y = x^3 - 2x + 3$
7	The gradient function of a curve which passes through $A(1, -12)$ is $3x^2 - 6x$ . Find the equation of the curve. <span style="float: right;">SPM 2004, K2</span>	8.	The gradient function of a curve which passes through $B(1, 5)$ is $3x^2 + 2$ . Find the equation of the curve.		
	$\frac{dy}{dx} = 3x^2 - 6x$ $y = \int (3x^2 - 6x) dx$ $= \frac{3x^3}{3} - \frac{6x^2}{2} + c$ $y = x^3 - 3x^2 + c$ $y = -12, x = 1, \quad -12 = 1^3 - 3(1) + c$ $-12 = -2 + c$ $c = -10$ Hence $\mathbf{y = x^3 - 6x - 10}$		$\frac{dy}{dx} = 3x^2 + 2$ $y = \int (3x^2 + 2) dx$ $=$		$y = x^3 + 2x + 2$
9.	The gradient function of a curve which passes through $P(1, -3)$ is $4x - 6$ . Find the equation of the curve.	10	The gradient function of a curve which passes through $Q(-1, 4)$ is $3x(x - 2)$ . Find the equation of the curve.		
			$y = 2x^2 - 6x + 1$		$y = x^3 - 3x^2 + 8$

## Skill 2 : Definite Integral

### Skill 2.1 : Integral of Algebraic Terms

$1. \int_1^2 2x \, dx = [x^2]_1^2$ $= 2^2 - 1^2$ $= 4 - 1$ $= 3$	$2. \int_0^3 4x^3 \, dx = [x^4]_0^3$ $=$	$3. \int_1^2 6x^2 \, dx =$ $=$
	[81]	[14]
$4. \int_1^2 3x^{-2} \, dx =$ $=$	$5. \int_1^3 \left(\frac{2}{x^3}\right) \, dx =$ $=$	$6. \int_1^2 \frac{3}{2x^2} \, dx =$ $=$
[ $\frac{3}{2}$ ]	[ $\frac{8}{9}$ ]	[ $\frac{3}{4}$ ]
$7. \int_0^3 (2 - 6x) \, dx =$	$8. \int_1^3 (4x - 3x^2) \, dx =$	$9. \int_0^3 x(2x + 1) \, dx =$
[-21]	[-10]	[22.5]
$10. \int_1^2 (2x - 1)(2x + 1) \, dx$ $=$	$11. \int_1^3 (3x - 2)^2 \, dx$ $=$	$12. \int_0^1 x(3x - 2) \, dx =$ $=$
[ $\frac{25}{3}$ ]	[38]	[1]

**Skill 2.2 : Definite Integral of  $\int_a^b (ax+b)^n dx$  ,  $n \neq -1$**

<p style="text-align: center;">E X A M P L E</p>	$\int_0^1 (3x+2)^4 dx = \left[ \frac{(3x+2)^5}{5 \cdot 3} \right]_0^1$ $= \left[ \frac{(3x+2)^5}{15} \right]_0^1$ $= \frac{5^5}{15} - \frac{2^5}{15}$ $= 206.2$	<p style="text-align: center;">Y o u  T r y !</p> $\int_1^2 \frac{3}{(2x-1)^2} dx = \int_1^2 3(2x-1)^{-2} dx$ $=$ $=$
<p>1.</p>	$\int_0^1 16(2+4x)^3 dx =$	<p>2.</p> $\int_0^1 6(x+2)^{-3} dx =$
<p>3.</p>	$\int_1^2 \frac{6}{(2x-1)^2} dx =$	<p>4.</p> $\int_2^3 \frac{24}{(3x-5)^3} dx =$

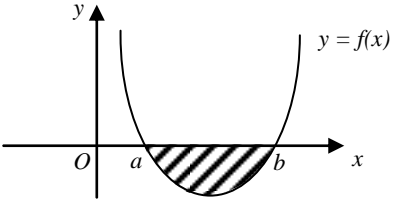
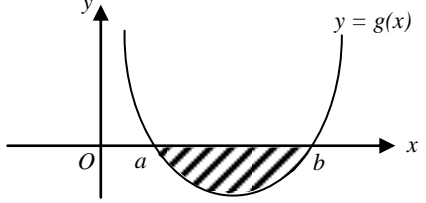
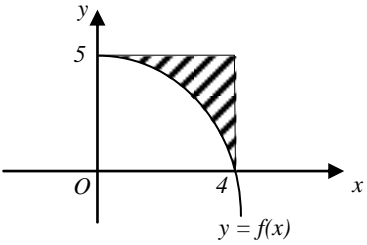
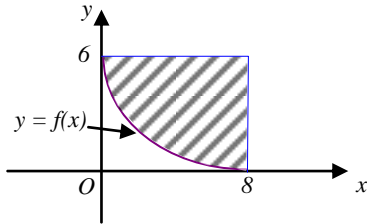
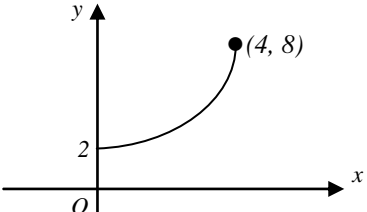
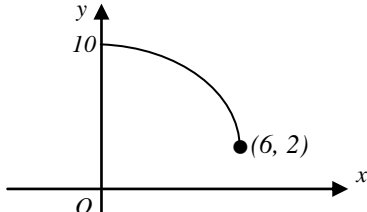
### Skill 2.3 : Applications of Definite Integral

<p>Given that <math>\int_1^3 f(x)dx = 4</math> , <math>\int_1^3 g(x)dx = 6</math> . Find the value of .</p>	
<p>1. <math>\int_3^1 f(x)dx =</math></p> <p style="text-align: right;">-4</p>	<p>2. <math>\int_1^3 [2 + f(x)]dx =</math></p> <p style="text-align: right;">8</p>
<p>3. <math>\int_1^3 [4f(x) - 2x]dx =</math></p> <p style="text-align: right;">8</p>	<p>4. <math>\int_1^3 \left[ \frac{3f(x) - 1}{2} \right] dx =</math></p> <p style="text-align: right;">5</p>
<p>5. <math>\int_1^3 [3f(x) - 2g(x)]dx =</math></p> <p style="text-align: right;">0</p>	<p>6. <math>\int_1^3 \left[ 2g(x) + \frac{1}{2}f(x) \right] dx =</math></p> <p style="text-align: right;">14</p>
<p>7 Given <math>\int_1^2 f(x)dx = 3</math> and <math>\int_3^2 f(x)dx = -7</math> . Find a) <math>\int_1^3 [5f(x) - 1]dx</math> b) the value of k if <math>\int_1^2 [kx - f(x)]dx = 8</math></p> <p style="text-align: right;">(a) 48 (b) <math>k = \frac{22}{3}</math></p>	<p>8 Given that <math>\int_0^4 f(x)dx = 3</math> and <math>\int_0^4 g(x)dx = 5</math> . Find a) <math>\int_0^4 f(x)dx \cdot \int_4^0 g(x)dx</math> b) <math>\int_0^4 [3f(x) - g(x)]dx</math></p> <p style="text-align: right;">(a) - 15 (b) 4</p>



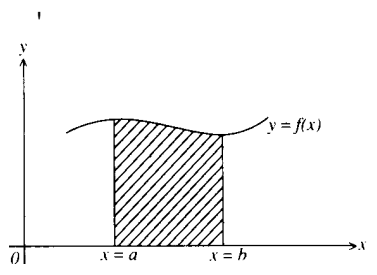
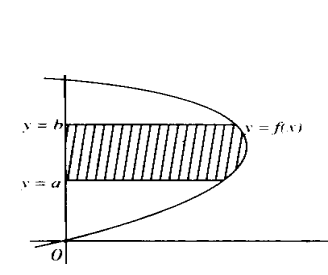
## Skill 2.4 : SPM Questions

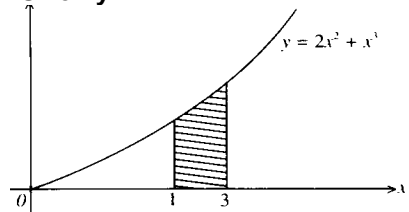
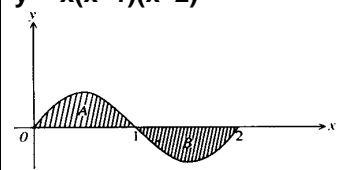
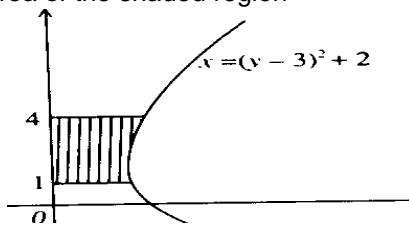
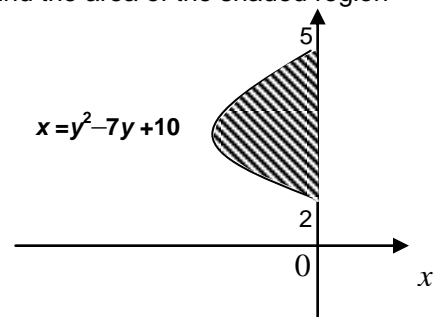
<p>1. Given that <math>\int_{-1}^k (2x-3)dx = 6</math>, where <math>k &gt; -1</math>, find the value of <math>k</math>. SPM 2004</p> <p style="text-align: right;">4</p>	<p>2. Given that <math>\int_0^k (2x-1)dx = 12</math>, where <math>k &gt; 0</math>, find the value of <math>k</math>.</p> <p style="text-align: right;">4</p>
<p>3. Given that <math>\int_0^k (3-4x)dx = -20</math>, where <math>k &gt; 0</math>, find the value of <math>k</math>.</p> <p style="text-align: right;">4</p>	<p>4. Given that <math>\int_0^k (6x+1)dx = 14</math>, where <math>k &gt; 0</math>, find the value of <math>k</math>.</p> <p style="text-align: right;">2</p>
<p>5. Given that <math>\int_2^6 f(x) = 7</math> and <math>\int_2^6 (2f(x) - kx) = 10</math>, find the value of <math>k</math>. SPM 2005</p> <p style="text-align: right;"><math>\frac{1}{4}</math></p>	<p>6. Given that <math>\int_1^5 g(x)dx = 8</math>, find</p> <p>(a) the value of <math>\int_5^1 g(x)dx</math>,</p> <p>(b) the value of <math>k</math> if <math>\int_1^5 [kx - g(x)]dx = 10</math>. SPM 2006</p> <p style="text-align: right;">(a) -8 (b) <math>\frac{3}{2}</math></p>

7.	<p>Diagram shows the curve <math>y = f(x)</math> cutting the x-axis at <math>x = a</math> and <math>x = b</math>. SPM 2006</p>  <p>Given that the area of the shaded region is <math>5 \text{ unit}^2</math>, find the value of <math>\int_a^b 2f(x)dx</math>.</p> <p style="text-align: right;">Answer : .....</p>	8.	<p>Diagram 9 shows the curve <math>y = f(x)</math> cutting the x-axis at <math>x = a</math> and <math>x = b</math>.</p>  <p>Given that the area of the shaded region is <math>6 \text{ unit}^2</math>, find the value of <math>\int_b^a 3g(x)dx</math>.</p> <p style="text-align: right;">Answer : .....</p>
9.	<p>Diagram shows part of the curve <math>y = f(x)</math>.</p>  <p>Given that <math>\int_0^4 f(x)dx = 15 \text{ unit}^2</math>, find the area of the shaded region.</p> <p style="text-align: right;">Answer : .....</p>	10	<p>Diagram shows part of the curve <math>y = f(x)</math>.</p>  <p>Given that the area of the shaded region is <math>40 \text{ unit}^2</math>, find the value of <math>\int_0^8 f(x)dx</math>.</p> <p style="text-align: right;">Answer : .....</p>
11.	<p>Diagram shows the sketch of part of a curve. (SPM 2001)</p>  <p>(a) Shade, on the given diagram, the region represented by <math>\int_2^8 x dy</math>.</p> <p>(b) Find the value of <math>\int_0^4 y dx + \int_2^8 x dy</math></p> <p style="text-align: right;">Answer : (b) .....</p>	12	<p>Diagram shows the sketch of part of a curve.</p>  <p>(a) Shade, on the given diagram, the region represented by <math>\int_2^{10} x dy</math>.</p> <p>(b) If <math>\int_2^{10} x dy = p</math>, find, in terms of <math>p</math>, the value of <math>\int_0^6 y dx</math>.</p> <p style="text-align: right;">Answer : (b) .....</p>

13.	<p>Given that <math>\int \frac{5}{(1+x)^4} dx = k(1+x)^n + c</math>, find the value of k and n. <span style="float: right;">SPM20'03</span></p> <p style="text-align: right;"><math>[k = -\frac{5}{3}, n = -3]</math></p>	14	<p>Given that <math>\int \frac{12}{(3x-2)^3} dx = k(3x-2)^n + c</math>, find the value of k and n.</p> <p style="text-align: right;"><math>k = -2, n = -2</math></p>
15.	<p>Given that <math>\int_0^1 (16x^2 + 10kx + k^2) dx = \frac{4}{3}</math>. Find the possible values of k. <span style="float: right;">(SPM 2002)</span></p> <p style="text-align: right;"><math>k = -1, -4</math></p>	16	<p>Given that <math>\int_0^1 (3x^2 + 10kx + 4) dx = 0</math>. Find the value of k.</p> <p style="text-align: right;"><math>k = -1</math></p>
17.	<p>(SPM 01) Given that <math>\frac{d}{dx} \left( \frac{x^2}{x-1} \right) = g(x)</math>, find the value of <math>\int_2^3 (2x - g(x)) dx</math>.</p> <p style="text-align: right;"><math>\frac{9}{2}</math></p>	18	<p>Given that <math>\frac{d}{dx} \left( \frac{x}{x-1} \right) = f(x)</math>, find the value of <math>\int_2^3 (4x + f(x)) dx</math>.</p> <p style="text-align: right;"><math>13\frac{1}{2}</math></p>

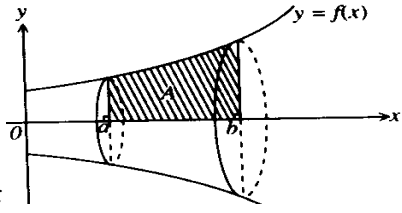
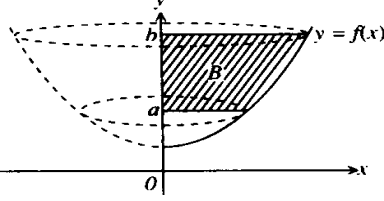
### Skill 3 : Find the Area of the region between a curve and the axes

 <p style="text-align: center;"> <math display="block">\text{Area} = \int_{y=a}^{y=b} x dy</math> </p> <p style="text-align: center;"> <b>The shaded area between the curve <math>y=f(x)</math>, <math>x = a</math>, <math>x = b</math> and the <math>x</math>-axis</b> </p>	 <p style="text-align: center;"> <math display="block">\text{Area} = \int_{x=a}^{x=b} y dx</math> </p> <p style="text-align: center;"> <b>The shaded area between the curve <math>y = f(x)</math>, <math>y = a</math>, <math>y = b</math> and the <math>y</math>-axis</b> </p>
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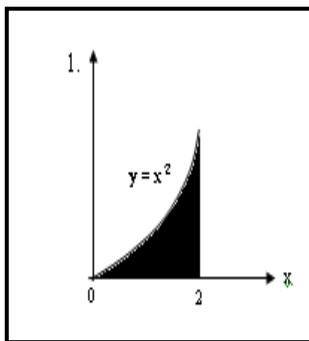
<p>1. Calculate the area of the shaded region . Given <math>y = 2x^2 + x^3</math></p>  <p style="text-align: right; margin-top: 20px;"><math>37 \frac{1}{3} \text{ unit}^2</math></p>	<p>2. Find the area enclosed by the curve <math>y = x(x-1)(x-2)</math></p>  <p style="text-align: right; margin-top: 20px;"><math>\frac{1}{2} \text{ unit}^2</math></p>
<p>3. Find the area of the shaded region</p>  <p style="text-align: right; margin-top: 20px;"><math>9 \text{ unit}^2</math></p>	<p>4. Find the area of the shaded region</p>  <p style="text-align: right; margin-top: 20px;"><math>4 \frac{1}{2} \text{ unit}^2</math></p>

**Homework : Text Book – Exercise 3.2.2 page 72 no 17**

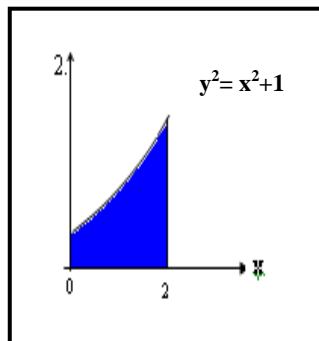
### Skill 4 : Find the Volume of Revolution

<p>The volume of revolution <math>V</math>, generated when the area under a curve <math>y = f(x)</math> by <math>x</math>-axis from <math>x = a</math> to <math>x = b</math> is rotated about the <math>x</math>-axis</p>  <p><math>I_x = \pi \int_a^b y^2 dx</math></p>	<p>The volume of revolution <math>V</math>, generated when the area under a curve <math>y = f(x)</math> by <math>x</math>-axis from <math>x = a</math> to <math>x = b</math> is rotated about the <math>y</math>-axis</p>  <p><math>I_y = \pi \int_a^b x^2 dy</math></p>
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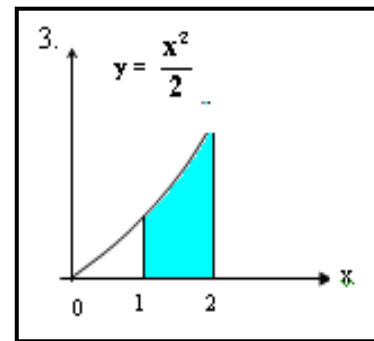
Calculate the volume generated when the shaded region is revolved  $360^\circ$  about the **x-axis**



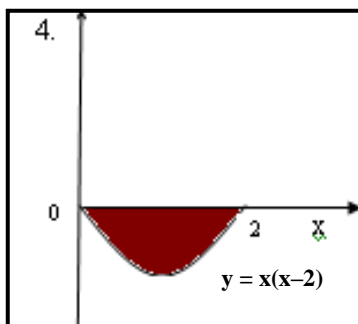
$$6\frac{2}{5}\pi$$



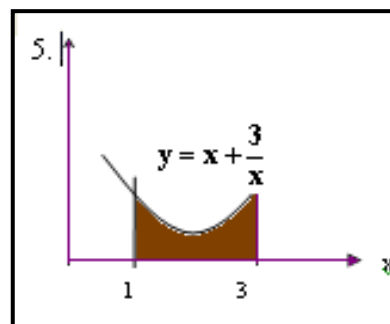
$$13\frac{11}{15}\pi$$



$$1\frac{11}{20}\pi$$

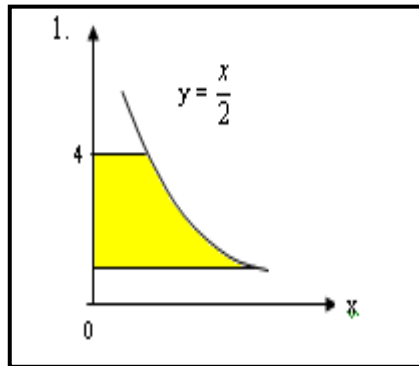


$$1\frac{1}{15}\pi$$

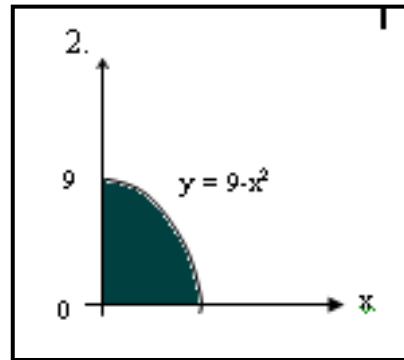


$$26\frac{2}{3}\pi$$

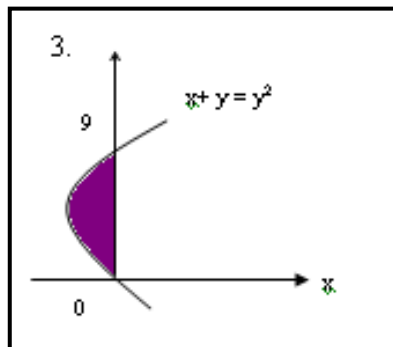
Calculate the volume generated when the shaded region is revolved  $360^\circ$  about the **y-axis**



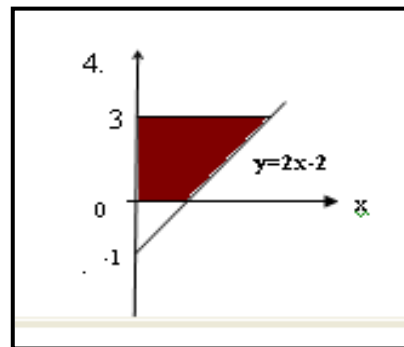
$3\pi$



$40\frac{1}{2}\pi$



$\frac{1}{\pi}$



$\frac{31}{3}\pi$

**Homework : Text Book – Exercise 3.2.2 page 72 no 18 and 19**

	SPM 1993	SPM 1998
1.	<p>Calculate the volume of the solid generated when the shaded region in the diagram is revolved through <math>360^\circ</math> about the y-axis</p> <p style="text-align: right;"><math>\frac{1}{2}\pi</math></p>	<p>2. Diagram below shows the graph of <math>y = x^2 - 2</math> and straight line <math>\frac{x}{3} - \frac{y}{5} = 1</math> Calculate the volume of the solid generated when the shaded region in the diagram is revolved through <math>360^\circ</math> about the y-axis</p> <p style="text-align: right;"><math>13\pi</math></p>

